**Functional**: If xRy and xRz, then y=z. If x has birthdate y and x has birthdate z, then y=z.

**Inverse Functional**: If xRy and zRy, then x=z. If x has social security number y and z has social security number y, then x=z.

**Transitive**: If xRy and yRz, then xRz . If x is contained in y and y is contained in z, then x is contained in z.

**Symmetric**: If xRy, then yRx. If x is a friend of y, then y is a friend of x.

**Asymmetric:** If xRy, then it is not the case that yRx If x is the parent of y, then it is not the case that y is the parent of x.

**Reflexive**: xRx, x is as tall as itself.

**Irreflexive**: It is not the case that xRx. No x is taller than itself

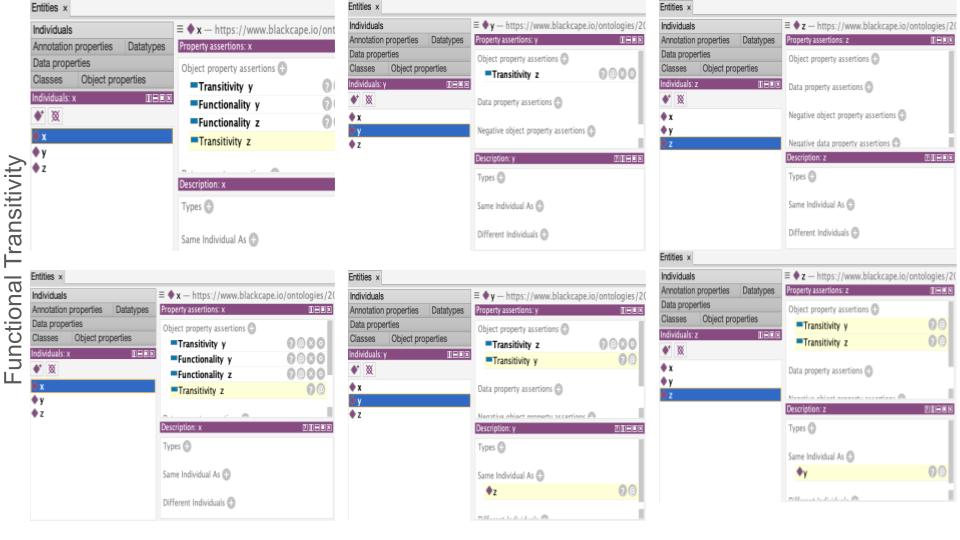
Suppose R is both symmetric and asymmetric. Then by symmetry for any x and y, if xRy it follows that yRx. However, by asymmetry it also follows that it is not the case that yRx. Hence, R cannot be both symmetric and asymmetric. Similarly, to explain why R cannot be both transitive and inverse functional, your explanation may take the form:

**Functional Transitivity (XNS)**

Suppose property R is both functional and transitive.

By functionality, for any x, y, z: if xRy ⋀ xRz, then it follows that y=z, and by transitivity if xRy ⋀ yRz then it also must follow that xRz.

Consider the case when y ≠ z, then by transitivity ( xRy ⋀ yRz ⇒ xRz ) and by functionality, [ xRy ⋀ xRz ) ⇔ y=z ] , but since y≠z and y=z cannot both be true, we have a contradiction. ⟂



**Inverse-Functional Transitivity (XNS)**

Suppose property R is both inverse functional and transitive.

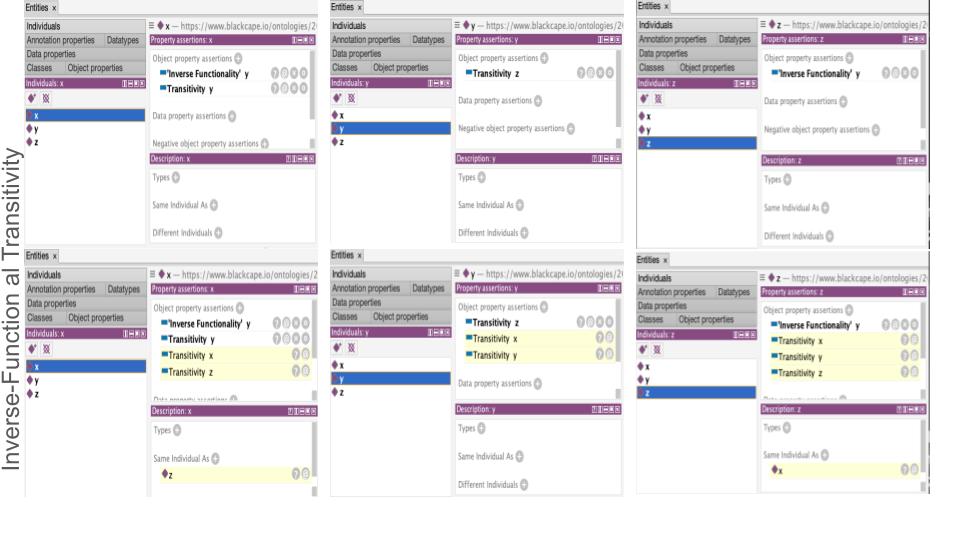
If R is Inverse Functional, then (xRy ⋀ zRy) ⇔ x=z

If R is Transitive, then ( xRy ⋀ yRz ) ⇒ xRz

\* Note that ff R is Reflexive, then ( xRy ⋀ yRx) and we know from the initial state of the table that Transitive Reflexivity is satisfiable.

By inverse functionality, If xRy ⋀ zRy, then x=z and by transitivity, if xRy ⋀ yRz then it follows that xRz.

Consider the case when x ≠ z, then then ( xRy ⋀ yRz ⇒ xRz ) holds true but Inverse functionality fails due to contradiction since it is obviously not the case that (x ≠ z) and (x = z). ⟂



Asymmetric Transitivity **(XNS)**

Irreflexive Transitivity **(XNS)**

Asymmetric Symmetry **(XUNSAT)**

Asymmetric Reflexivity **(XUNSAT)**

Irreflexive Reflexivity **(XUNSAT)**